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14. ABSTRACT We address several stochastic control problems related to queueing networks with fractional Brownian motion (fBM) input are addressed. Abelian limit relationships among the value functions of these control problems are also established. These problems are motivated by fluid networks related to on-off queueing systems. A stochastic comparison theorem for stochastic differential equations driven by fBM is obtained. We compute optimal strategies for a class of goal problems when the state process is corrupted by a fBM noise.					
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Report Title

Stochastic control problems for processes with long-range dependence & internet traffic

ABSTRACT

We address several stochastic control problems related to queueing networks with fractional Brownian motion (fBM) input are addressed. Abelian limit relationships among the value functions of these control problems are also established. These problems are motivated by fluid networks related to on-off queueing systems. A stochastic comparison theorem for stochastic differential equations driven by fBM is obtained. We compute optimal strategies for a class of goal problems when the state process is corrupted by a fBM noise.

In the second part, we address several issues related to controlled queueing networks with impatient customers in heavy traffic. We obtain heavy traffic approximations for queue length and offered waiting time. We also address a control problem for a queueing network in heavy traffic with impatient customers. Such results are applicable for internet traffic and for the dynamics of large telephone call centers.

List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Optimal control of a stochastic processing system driven by fractional Brownian motion input

Number of Papers published in peer-reviewed journals: 1.00

(b) Papers published in non-peer-reviewed journals or in conference proceedings (N/A for none)

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(c) Presentations

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1. Lee, C. and Weerasinghe, A. Stationarity and control of a random fluid network with fractional Brownian motion input.
2. Lee, C. and Weerasinghe, A. Convergence of a queueing system in heavy traffic with general abandonment distributions
3. Comparison and control of processes driven by fractional Brownian motion.
4. Abandonment vs. blocking in a controlled many server queueing system with impatient customers

Number of Manuscripts: 4.00

Patents Submitted

Patents Awarded

Awards

Graduate Students

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
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Total Number:	

Names of Post Doctorates

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Names of Faculty Supported

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Names of Under Graduate students supported

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This section only applies to graduating undergraduates supported by this agreement in this reporting period

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Final Report

Ananda Weerasinghe*

December 22, 2010

1 Statement of the Problem

In the first part of this project, we study several stochastic control problems driven by one-dimensional fractional Brownian motion (fBM) process. Such processes are non-Markovian and their future behavior may depend on the history. These models arise in many practical situations and our work is motivated by approximating large on-off queueing networks by fBM driven stochastic systems. We address several stochastic optimal control problems for fBM driven models. We obtain existence theorems for optimal strategies. In the case of long-run average cost optimization problems, due to highly non-Markovian nature of the fBM, it is not apriori known whether the initial data plays a role in the overall cost and hence for the choice of an optimal strategy. We answer this problem by showing that the long-run cost is independent of the initial data and the tool we developed here to obtain this result is a coupling time argument and it may have other applications. Stochastic control of models driven by fBM is a new research area and our contributions will help further development of this area and also to understand the behavior of controlled large on-off queueing systems. It is well known that stochastic comparison theorems are very useful in the analysis of control problems driven by the ordinary Brownian motion. Here we develop such comparison results for models driven by fBM and they can be used in finding optimal strategies for stochastic control problems.

In the second part, we address several control problems related to stochastic networks in heavy traffic, such as internet traffic, telephone call centers with large number of servers and large processing networks. In a queueing system with a finite waiting room capacity in heavy traffic, customers tend to abandon the queue if the delay is too long. On the other hand, if the waiting room is too small, it will be full most of the time and customers are rejected often. Thus a system manager will face the problem of appropriate waiting room size and it is preferred to choose this waiting room capacity in a dynamic, history-dependent fashion leading to non-Markovian strategies. We formulate this as an optimal control problem and derive a threshold type optimal strategy. We also investigate the behavior of offered waiting times for impatient customers in a queueing system in heavy traffic and establish a new class of diffusion approximations.

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2 Controlled queueing networks with fractional Brownian motion input.

In a recently published article [1], we have analyzed the control of a single server queueing model with a fractional Brownian motion (fBM) input. I was the main author of this article. In this model, for a given initial value $x \geq 0$ and a fixed control variable $u \geq 0$, the controlled state process X_x^u is defined by

$$X_x^u(t) = x - ut + \sigma(u)W_H(t) + L_x^u(t), \quad t \geq 0, \quad (2.1)$$

where the process L_x^u is given by

$$L_x^u(t) = -\min\left\{0, \min_{s \in [0, t]} (x - us + \sigma(u)W_H(s))\right\}, \quad t \geq 0. \quad (2.2)$$

From the above equations, it follows that $X_x^u(t) \geq 0$ for all $t \geq 0$. Notice that the process L_x^u has continuous paths, and it increases at times when $X_x^u(t) = 0$. The processes X_x^u and L_x^u are both adapted to the fBM filtration. The function σ is continuous and takes only positive values. The process X_x^u can be thought of as the queue-length of a single-server queueing system with a fBM input and in such a situation, the control variable u is related to the deterministic service rate.

Our work is motivated by the control problems associated with a discrete queueing network model related to internet traffic data (ON-OFF processes). Such discrete control problems are not easily tractable. But, under heavy traffic conditions as describe in [1], suitably scaled queue length process converges in distribution to the above model (2.1). This can be verified using the fundamental theorem for ON-OFF processes as given in [5]. For details, see [1]. Therefore, the analysis of the control problems related to our model enables us to understand asymptotically optimal strategies for the corresponding controlled discrete queueing networks in heavy traffic. We have analyzed three types of control problems related to this model as described below:

A. Ergodic Control problem.

We consider the minimization of the long run average cost functional

$$I(u, x) := \limsup_{T \rightarrow \infty} \frac{1}{T} E \left(\int_0^T [h(u) + C(X_x^u(t))] dt + \int_0^T p dL_x^u(t) \right), \quad (2.3)$$

where h and C are non-negative cost functions defined on $[0, \infty)$ and p is a non-negative constant. These cost functions are both continuous and increasing to infinity. The function h represents the control cost and C represents the congestion cost associated with the system. The constant $p \geq 0$ is the cost per unit time when the workload in the system is empty. The value function is defined by

$$V_0(x) = \inf_{u \geq 0} I(u, x). \quad (2.4)$$

In [1], we have obtained the following results:

Theorem 2.1.

- (i) The functional $I(u, x)$ in (2.3) is independent of the initial point x and has the representation $I(u) \equiv I(u, x) = h(u) + pu + G(u)$.
Here the function $G(u)$ is given by $G(u) = E[C(Z_u)]$ where $Z_u := \max_{s \geq 0} \{W_H(s) - us\}$.
- (ii) The value function $V_0(x)$ is independent of x and there is an optimal control $u^* > 0$ such that $I(u^*) = V_0(x)$ for all x .

Proof of this theorem involves a coupling argument of the above state process X_x^u with a stationary process. and we believe that our coupling method will be useful in the analysis of the problems related to the long term behavior of X_x^u .

B. A constrained minimization problem.

We can apply our results in the previous theorem to obtain an optimal control for the following constrained minimization problem driven by a fBM model. We are not aware of any other constrained optimization problems for models driven by fBM available in the literature. Our model here is of the form

$$Y^u(t) = x - ut + \sigma(u)W_H(t) + K^u(t). \quad (2.5)$$

We keep the initial point x fixed throughout. Here σ is a non-negative continuous function, $K^u(\cdot)$ is a non-negative non-decreasing right-continuous with left limits (RCLL) process adapted to the natural filtration $(\mathcal{F}_t)_{t \geq 0}$, where \mathcal{F}_t is the σ -algebra generated by $\{W_H(s) : 0 \leq s \leq t\}$ augmented with all the null sets. Furthermore, $K^u(0) = 0$ and the process K^u is chosen by the controller in such a way that the state process Y^u is constrained to non-negative reals. In this situation, the controller is equipped with two controls: the choice of $u > 0$ and the choice of K^u process subject non-negativity of the Y^u process. The initial point x is fixed.

Let $m > 0$ be any fixed positive constant. The constrained minimization problem is the following:

Minimize

$$\limsup_{T \rightarrow \infty} \frac{1}{T} E \left(\int_0^T [h(u) + C(Y^u(t))] dt \right) \quad (2.6)$$

Subject to:

$$\limsup_{T \rightarrow \infty} \frac{E(K^u(T))}{T} \leq m. \quad (2.7)$$

We have the following result:

Theorem 2.2. Let u_0^* be the optimal control obtained in Theorem 1.1 corresponding to the case $p = 0$. Introduce $u^*(m)$ by $u^*(m) = \min\{u_0^*, m\}$.

Then for each $m > 0$ fixed, the optimal state process is given by $X_x^*(t) = x - u^*(m)t + \sigma(u)W_H(t) + L_m^*(t)$ where L_m^* is the corresponding “local-time process” associated with X_x^* as similar to (2.1). Hence, the pair $(u^*(m), L_m^*)$ is an optimal strategy and it is independent of the initial point x .

C. Abelian limit relationships.

In relation with the cost functional in (2.3), we also introduce the corresponding finite-horizon cost minimization problem on an interval $[0, T]$ and the infinite-horizon discounted cost minimization problem. Let the value functions of these two problems be given by $V(x, T)$ and $V_\alpha(x)$ respectively. Here $\alpha > 0$ is the discount factor in the infinite horizon discounted cost minimization problem. We obtain the existence of an optimal control for each of these problems and establish the Abelian limit relationships among the value functions $V_0(x)$, $V_\alpha(x)$ and $V(x, T)$ as described in the following theorem.

Theorem 2.3. $\lim_{\alpha \rightarrow 0^+} \alpha V_\alpha(x) = \lim_{T \rightarrow \infty} \frac{V(x, T)}{T} = V_0(x)$ for all x .

In the proof of this theorem, we also show that the convergence of the optimal controls of $V_\alpha(x)$ (similarly, $V(x, T)$) as α tends to zero (as T tends to infinity), to the optimal control of $V_0(x)$.

F. Tandem Fluid Networks.

In a recently completed article [3], we have generalized the results of [1] to tandem fluid networks in heavy traffic. We consider a controlled two station tandem fluid network with heavy tailed on-off sources. Under appropriate heavy traffic conditions, we establish the weak convergence of the two-dimensional workload process to a controlled queueing model driven by a two dimensional fBm input. We also allow the components of the input fBm to be correlated with a constant correlation coefficient. For this model, first we show the existence of a stationary distribution for the work load process with zero initial data. Then, we show that there exists a stationary work load process on the same probability space so that its probability distribution is invariant over time. Then finally, under mild assumptions on the initial data, we show that any state process coalesce with the stationary state process and the corresponding coupling time is finite and has finite moments. We also identify the asymptotics of the tail of this stationary distribution. As an application of this result, we address a cost minimization problem for long-run average cost and show the existence of an optimal strategy which is independent of the initial data. In conclusion, we showed how to extend these results to a tandem queue with any finite number of stations and also obtained the corresponding stationary distribution.

3 Stochastic Comparison Theorems.

Stochastic comparison theorems are well known for the stochastic differential equations (SDE's) driven by ordinary Brownian motion. They have a wide range of applications in stochastic analysis and in particular, they play an important role in stochastic control theory. Such results are used to compare expected pay-off values from different admissible control strategies. In the case of SDE's driven by fBM, we have obtained the following results related to stochastic comparison theorems in a recent article ([6]). One should note that the stochastic integral in the SDE below is defined as a pathwise Riemann-Stieltjes integral and this SDE has a path wise unique solution under the assumptions listed below.

Theorem 3.1. *Let $H > \frac{1}{2}$, $T > 0$ and W_H be a fBM with Hurst parameter H . Consider two solutions X_i ($i = 1, 2$) to the SDE's driven by the same fBM, where*

$$X_i(t) = x_i + \int_0^t b_i(X_i(s))ds + \int_0^t \sigma(X_i(s))dW_H(s) \quad (3.1)$$

for all $0 \leq t \leq T$. For $i = 1, 2$, we assume that the functions b_i and σ are Lipschitz continuous and the derivative of σ is locally Hölder continuous, and $b_1(x) \geq b_2(x)$ for all x . If the initial conditions satisfy $x_1 > x_2$, then $X_1(T) \geq X_2(T)$ a.s. for all $T > 0$.

Next we prove the following more general version which has a control theoretic interpretation. Let b be a given Lipschitz continuous function. We consider a controlled state process X_x^u which satisfies

$$X_x^u(t) = x + \int_0^t u(s)ds + \int_0^t \sigma(X_x^u(s))dB_H(s) \quad (3.2)$$

where σ is a Lipschitz continuous function with Lipschitz constant $K > 0$ and its derivative is locally Hölder continuous. Here B_H is a fractional Brownian motion with Hurst parameter $H > \frac{1}{2}$ and is adapted to a filtration (\mathcal{F}_t) . The control process u is adapted to the filtration (\mathcal{F}_t) and $\int_0^t |u(s)|ds < \infty$ for all $t \geq 0$ a.s. We say u is an admissible control process, if it satisfies the condition

$$u(t) \leq b(X_x^u(t)) \quad \text{for all } t \geq 0 \quad (3.3)$$

and the corresponding state process X_x^u belongs to the function space $W_0^{\alpha, \infty}[0, T]$ for some $\alpha_0 > \alpha > 1 - H$. Here $\alpha_0 = \min\{\frac{1}{2}, \frac{\delta}{1+\delta}\}$, where $\delta > 0$. If u is an admissible control then we say X_x^u is an admissible state process. Let \mathcal{A}_x be the collection of all such admissible state processes X_x^u described above. Next we introduce a process Y_x which is driven by feed-back type control $b(Y_x(s))$ and it satisfies

$$Y_x(t) = x + \int_0^t b(Y_x(s))ds + \int_0^t \sigma(Y_x(s))dB_H(s). \quad (3.4)$$

Then, we have the following theorem:

Theorem 3.2. *Assume that the functions b and σ are Lipschitz continuous and the derivative of σ is locally Hölder continuous as in the previous theorem. Then there is an optimal process Y_x in \mathcal{A}_x such that*

$$Y_x(t) \geq X_x^u(t) \quad \text{a. s.}$$

for all $t \geq 0$, where X_x^u is any process in \mathcal{A}_x . Furthermore, Y_x satisfies the SDE

$$Y_x(t) = x + \int_0^t b(Y_x(s))ds + \int_0^t \sigma(Y_x(s))dB_H(s). \quad (3.5)$$

Hence the optimal control u^ is feed-back type and is described by $u^*(t) = b(Y_x(t))$ for all $t \geq 0$.*

Both of the above theorems are path-wise comparison results. We have also obtained the following mean comparison theorem when the Hurst parameter $0 < H < 1$.

Let b be a convex, Lipschitz continuous function. We consider a general state process X which satisfies the equation

$$X(t) = x + \int_0^t u(s)ds + \sigma B_H(t). \quad (3.6)$$

where σ is a constant, B_H is a fBM with Hurst parameter $0 < H < 1$ and is adapted to a filtration (\mathcal{F}_t) . The control process u is also adapted to the filtration (\mathcal{F}_t) , $\int_0^t |u(s)|ds < \infty$ for each $t \geq 0$ a.s. and it also satisfies

$$u(t) \leq b(X(t)), \quad (3.7)$$

for all $t \geq 0$. Next, we introduce the process Y , which is a solution to the SDE given by

$$Y(t) = y + \int_0^t b(Y(s))ds + \rho W_H(t). \quad (3.8)$$

where ρ is a positive constant, W_H is a fBM with the same Hurst parameter $0 < H < 1$ in a (possibly a different) probability space $(\hat{\Omega}, \hat{\mathcal{F}}, \hat{P})$ and W_H is adapted to a filtration $(\hat{\mathcal{F}}_t)$. The Lipschitz continuity of b guarantees a strong solution to (3.8) with respect to any fBM W_H . Then we have the following theorem:

Theorem 3.3. *Let σ and $\rho > 0$ be constants which satisfy $|\sigma| \leq \rho$ and the initial conditions satisfy $x \leq y$. Then for any $T > 0$ and for any convex function h ,*

$$E[h(X(T))] \leq E[h(Y(T))].$$

In particular,
$$E[\max_{[0,T]} X(s)] \leq E[\max_{[0,T]} Y(s)].$$

All the above theorems are proved in ([6]).

4 Controlling a process to a goal.

A. Goal problems without fuel.

We have solved the following probability maximization problem in [6] for processes driven by fBM.

Consider a state process X_x^u given by

$$X_x^u(t) = x + \int_0^t u(s)ds + \sigma B_H(t) \quad (4.1)$$

where $\sigma > 0$ is a constant. Here B_H is a fractional Brownian motion with Hurst parameter $0 < H < 1$ and is adapted to a filtration (\mathcal{F}_t) . The control process u is adapted to the filtration (\mathcal{F}_t) , $\int_0^t |u(s)|ds < \infty$ and $u(t)$ belongs to $\mathcal{A}(X_x^u(t))$ for all $t \geq 0$ a.s. where each control set $\mathcal{A}(y)$ is a subset of \mathbf{R} and the collection $\{\mathcal{A}(y) : y \text{ in } \mathbf{R}\}$ is a

priori available to the controller.

Let a and b be constants so that $a < x < b$. We would like to consider the problem of choosing the constant σ and the drift control u to maximize the probability that the state process X_x^u reach the goal b before reaching a . In the case of the models driven by standard Brownian motion, solutions to such problems are already known.

Our main assumption here is that the existence of a positive constant $\rho > 0$ and a Lipschitz continuous function b defined on \mathbf{R} so that

$$b(y) = \sup \mathcal{A}(y) \quad (4.2)$$

and $\rho > 0$ satisfies (4.3) below. Admissibility of the control process u guarantees that $u(t) \leq b(X_x^u(t))$ for all $t \geq 0$.

We address this problem in the following two cases:

- (i) If $b(y) \geq 0$ for all y , then we assume that available σ are bounded below by a positive constant and we let $\rho = \inf \sigma$.
- (ii) If $b(y) \leq 0$ for all y , then we assume that the available σ are bounded above and in that case, we set $\rho = \sup \sigma$.

Therefore, in both cases

$$\frac{b(y)}{\rho^{\frac{1}{H}}} = \sup \frac{b(y)}{\sigma^{\frac{1}{H}}} \quad (4.3)$$

holds, where the sup is taken over all available constants $\sigma > 0$. Consider the process Y_x described by

$$Y_x(t) = x + \int_0^t b(Y_x(s))ds + \rho W_H(t). \quad (4.4)$$

where W_H is a fBM with the same Hurst parameter $0 < H < 1$ in a (possibly a different) probability space $(\hat{\Omega}, \hat{\mathcal{F}}, \hat{P})$ and W_H is adapted to a filtration $(\hat{\mathcal{F}}_t)$. Then we have the following theorem.

Theorem 4.1. *Assume (4.2) and (4.3). Let X_x^u be any state process which satisfies (4.1). Let Y_x be the process described above in (4.4). Then,*

$$P[X_x^u \text{ reaches } b \text{ before } a] \leq P[Y_x \text{ reaches } b \text{ before } a]. \quad (4.5)$$

B. Goal problems with fuel.

In [6], we also address the above goal problem in the presence of fuel available to the controller. The controller can spend the fuel to gain a deterministic displacement which is proportional to the amount of fuel spent. This is a singular control problem driven by fBM noise. In this situation, we introduce two extreme strategies known as *bold play* and *timid play* where in bold play strategy, controller spends all the fuel immediately and in timid play strategy, controller spends fuel only if the position is at origin to receive a forward push. In two theorems, we have established that bold play and timid play are optimal strategies under different circumstances.

5 Utility maximization.

We consider a controlled state process

$$X_x^u(t) = x + \int_0^t u(s)[\mu ds + dB_H(s)] \quad (5.1)$$

where W_H is a fractional Brownian motion with Hurst parameter $\frac{1}{2} < H < 1$ and is adapted to a filtration (\mathcal{F}_t) . The control process u is adapted to the filtration (\mathcal{F}_t) and $\int_0^t |u(s)|ds < \infty$. The initial point $x \geq 0$. The stochastic integral $\int_0^t u(s)dB_H(s)$ is considered a Wick type integral rather than the path-wise stieljes integral we have used in the comparison theorems.

The problem addressed here is motivated by the following continuous-time gambling scenario: Consider a gambler with initial wealth of $x > 0$ dollars and who would like to bet an amount of $u(t)$ dollars at time t in a continuous-time gambling scheme where the state process of the gamble is given by $\mu t + B_H(t)$. Gambler's bet size $u(t)$ at time t may depend on all the information up to time t . Hence, the gambler's fortune at time t is given by $X_x^u(t)$ of (5.1). In this context, to have the infinitesimal mean rate of change of fortune to be $\mu E[u(t)]dt$, it is natural to use wick-type stochastic integral $\int_0^t u(s)dB_H(s)$, since wick type integral satisfies $E \int_0^t u(s)dB_H(s) = 0$.

Gambler's objective is to maximize the expected utility of the terminal wealth $E[U(X_x^u(t))]$ where U is a concave utility function.

For this problem, we derived an explicit optimal strategy in [6] using the fractional Girsanov theorem and the fractional Clark-Ocone formula.

6 Networks in heavy traffic.

- A. **Offered waiting times** In a recent manuscript [4], we have analyzed a sequence of single-server queueing systems with impatient customers with general patience time distributions. We established the heavy traffic approximation for the scaled offered waiting time process and obtained a diffusion process as the heavy traffic limit. These results lead to a large class of diffusion processes with non-linear drift coefficients as such heavy traffic limits. As a consequence, we also obtained the same diffusion process for the heavy traffic limit of the scaled queue length process. We also showed the convergence of the expected value of an infinite-horizon discounted cost functional of the queueing system to that of the diffusion process under the heavy traffic limit.
- B. **Many server queueing systems.** This project is a joint work with Professor Avi Mandelbaum who is an expert in the theory of queueing systems in heavy traffic. Motivated by my previous work [2] on queueing systems in heavy traffic, we consider an important control problem which has applications to management of a large call center, a receiver of a wireless communications network or a communication system related to internet traffic. This work is recently submitted for publication and the current manuscript consists of 52 pages. We were able to make an important generalization to currently available results in the literature, where the system manager is allowed to

choose a non-Markovian blocking mechanism which may depend on the current state as well as the past history of the network.

We consider a controlled queueing system with many servers, finite-capacity queue with impatient customers. Arrivals are general; impatience and service times are exponential, and servers are iid. Customers who arrive to a full queue are lost, and customers who wait too long in the queue abandon. Motivated by profit maximization (or cost minimization) in moderate-to-large communication networks, blocking corresponds to a busy-signal and abandonment to a disconnect before being served. The problem is the tradeoff between these two: the larger the queue capacity the less the blocking which, in turn, leads to longer queues, more waiting and hence more abandonment. Therefore, we intend to introduce linear costs proportional to the queue length, the number of blocked and abandoning customers, and the number of idle servers. These costs are discounted over an infinite horizon to yield a stochastic control problem, in which queue capacity is one's control variable. We solve this problem asymptotically, in the Halfin-Whitt heavy-traffic regime. Specifically, we study cost minimization as the arrival rate, the number of servers and (hence) the queue-capacity increase indefinitely, at rates that give rise to what has been called the QED (Quality- and Efficiency-Driven) regime: relatively few blocking and abandonment (service-quality) jointly with high servers' utilization (service-efficiency). Associated with the original control problem, there is a diffusion control problem. We solve the latter explicitly and our solution led to a threshold type optimal strategy. We use this solution to construct asymptotically optimal controls for the controlled queueing network problem.

For this, we obtained moment bounds for the scaled queue length. Then, we were able to derive an explicit asymptotically-optimal queue-capacity, including infinite capacity (no busy-signals) if blocking costs far exceed abandonment costs. It turned out that the value function of the diffusion control problem is a twice continuously differentiable convex function. We show that the value function of the diffusion control problem provides an achievable asymptotic lower bound for the cost function of the original queueing problem.

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